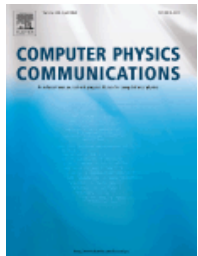
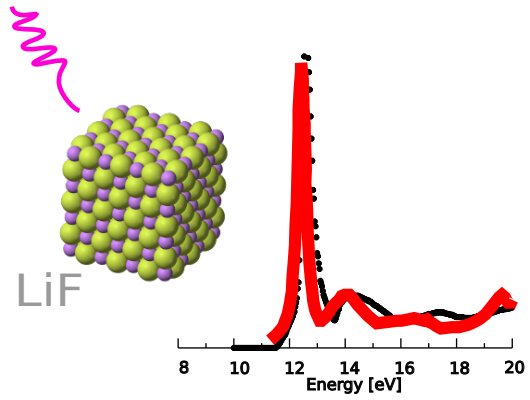


In real time: nonlinear optical spectroscopy

Myrta Grüning - Queen's University Belfast

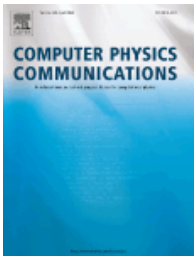
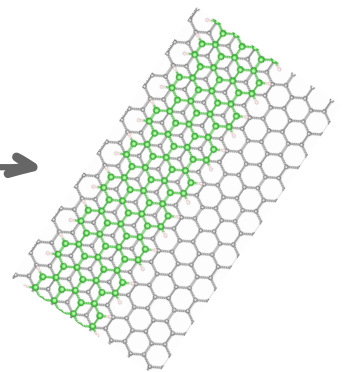
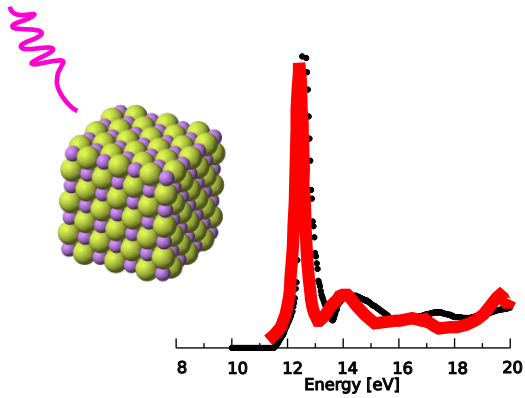
ab initio...



Marini et al.



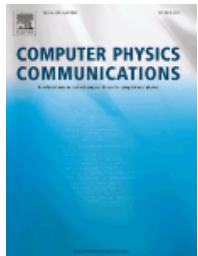
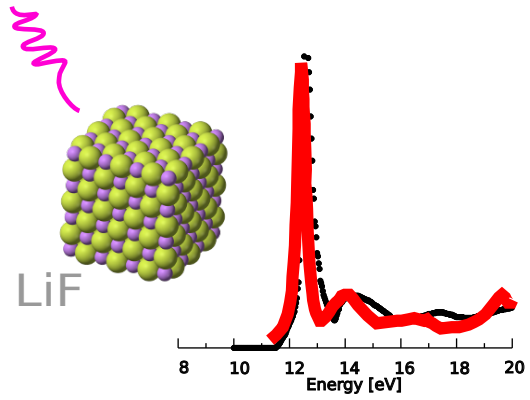
...towards larger systems



Marini et al.



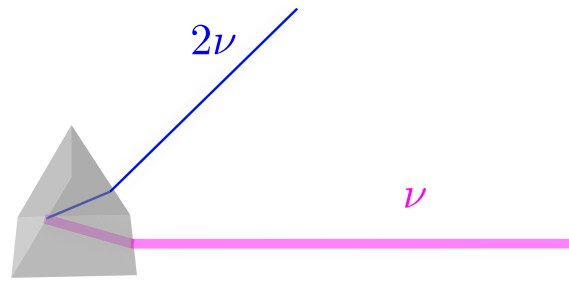
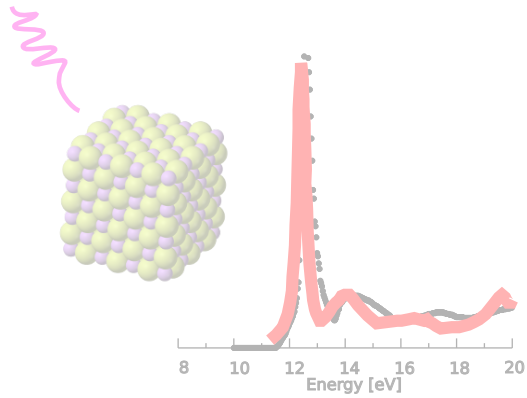
ab initio...



Marini et al.



...towards other spectroscopies

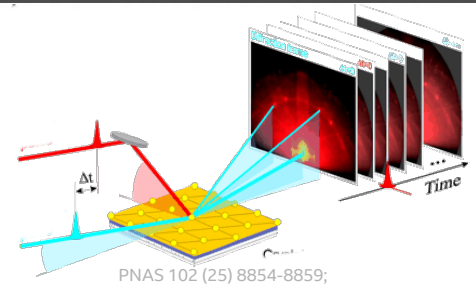
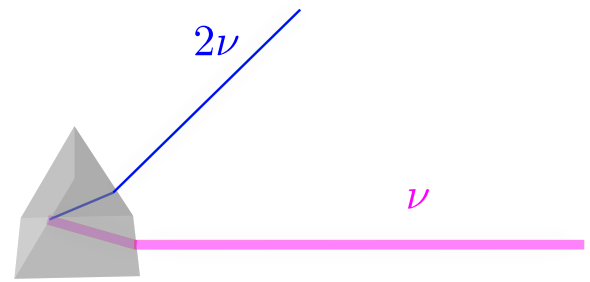
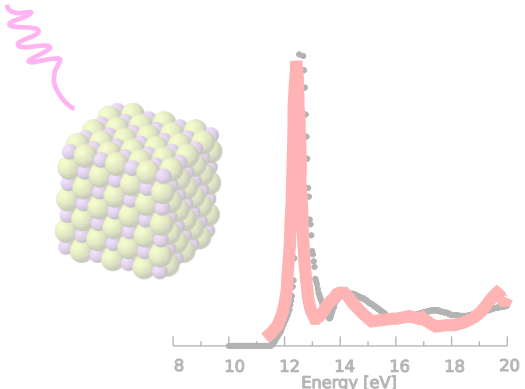


Nonlinear
optics

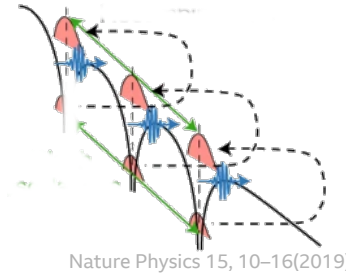
Linear
response

Nonlinear
regime

...towards other spectroscopies



pump-probe



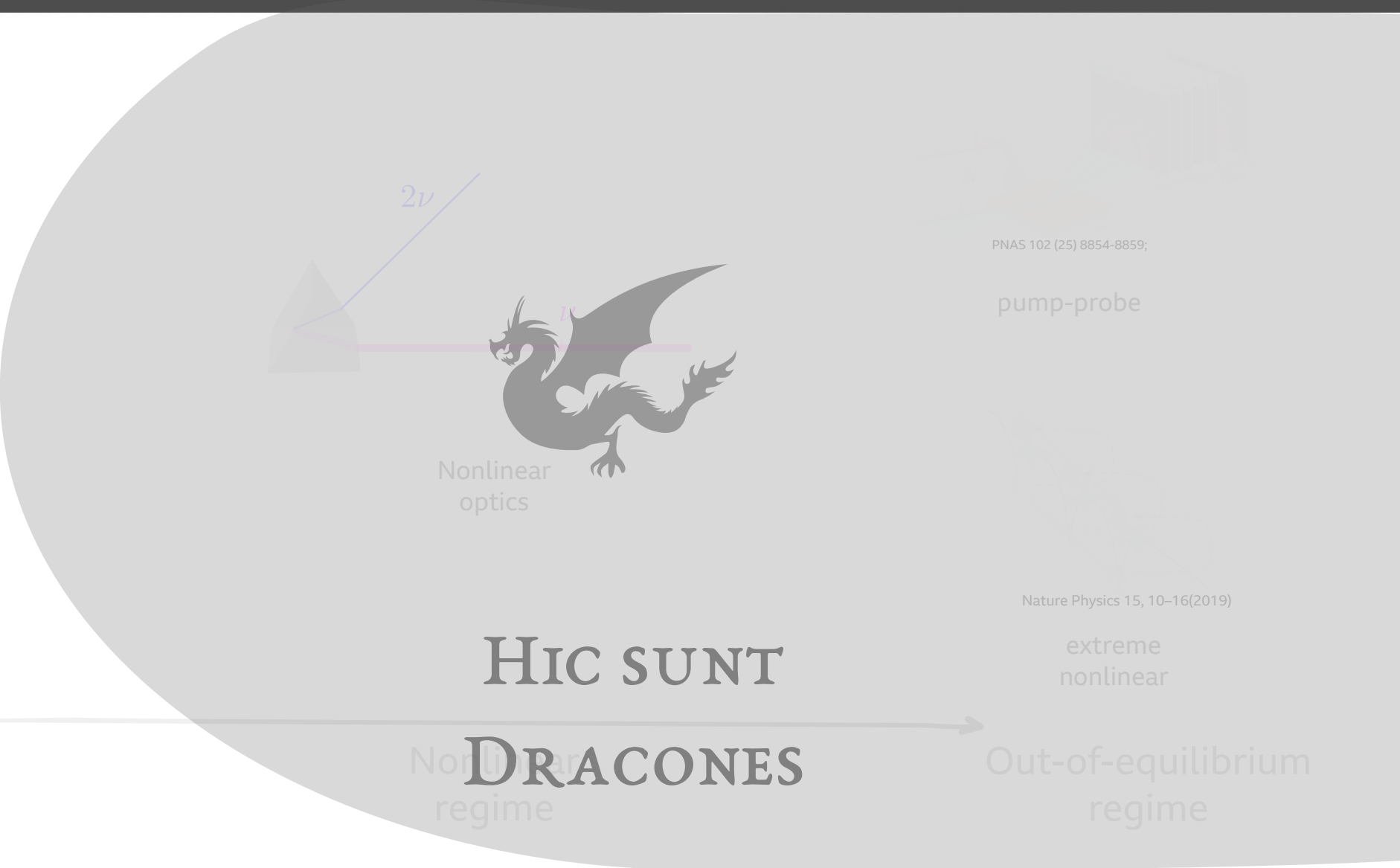
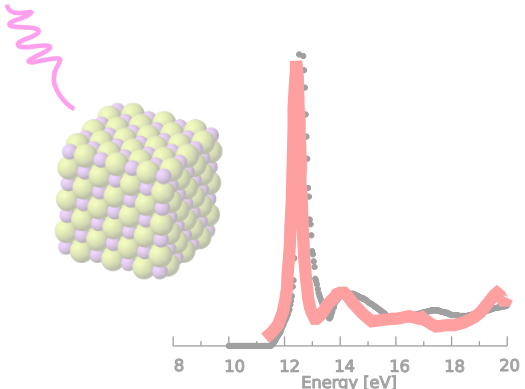
extreme nonlinear

Linear response

Nonlinear regime

Out-of-equilibrium regime

an uncharted territory



PNAS 102 (25) 8854-8859;

pump-probe

Nature Physics 15, 10-16(2019)

extreme
nonlinear

HIC SUNT

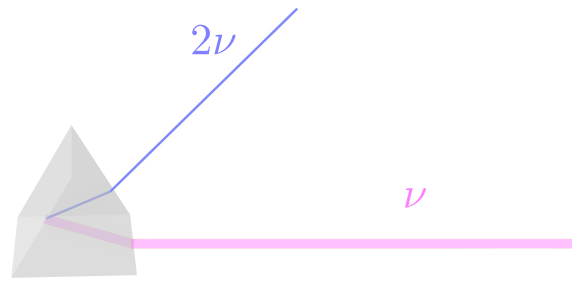
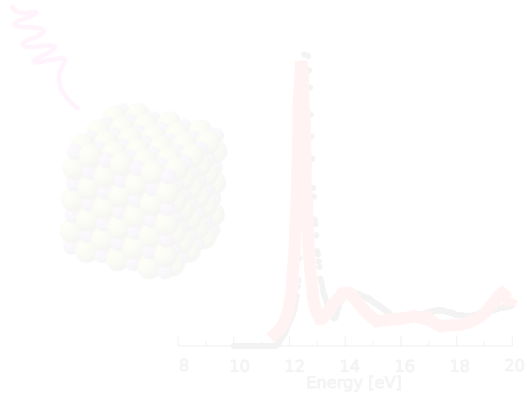
DRACONES

Linear
response

Nonlinear
regime

Out-of-equilibrium
regime

the dragons



HIGH ORDER E-H DIAGRAM



PNAS 102 (25) 8854-8859;

pump-probe



Nature Physics 15, 10-16(2019)

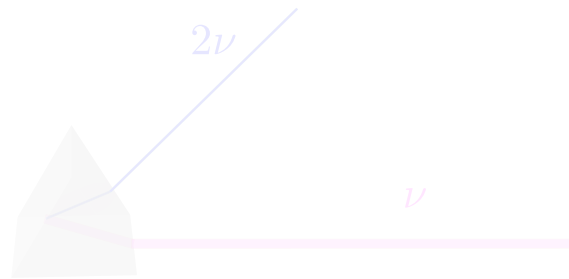
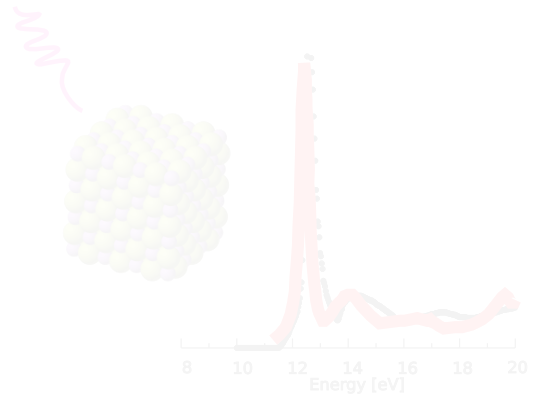
extreme
nonlinear

Linear
response

Nonlinear
regime

Out-of-equilibrium
regime

the dragons

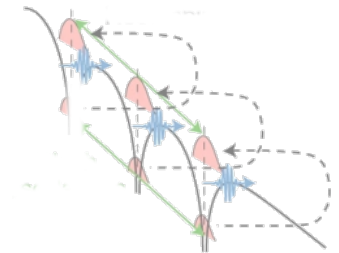


PNAS 102 (25) 8854-8859;

pump-probe



NON
PERTURBATIVE



Nature Physics 15, 10-16(2019)

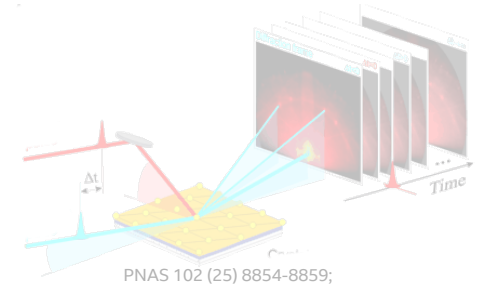
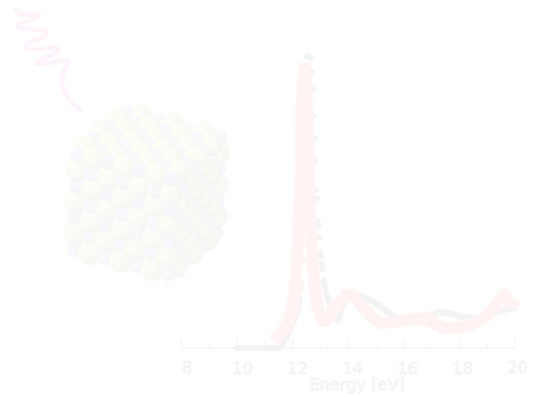
extreme
nonlinear

Linear
response

Nonlinear
regime

Out-of-equilibrium
regime

the dragons



pump-probe



extreme
nonlinear

Linear
response

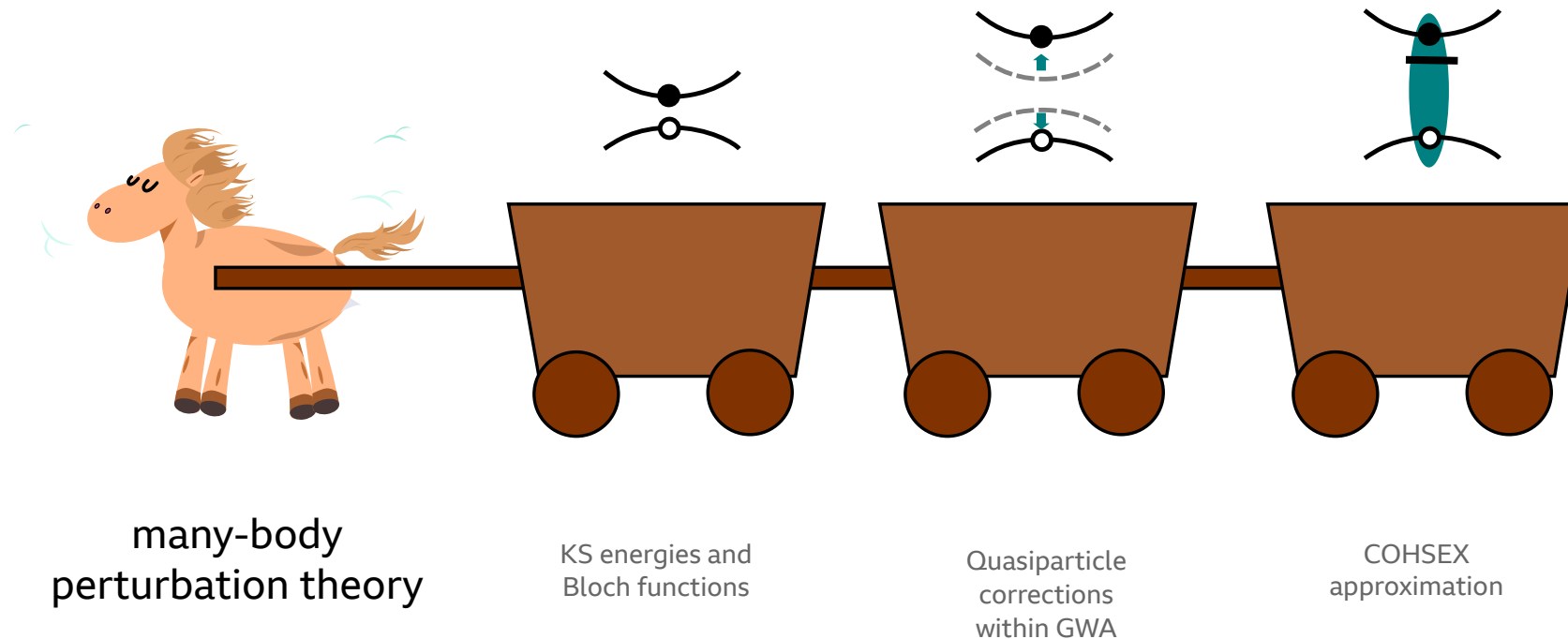
Nonlinear
regime

Out-of-equilibrium
regime

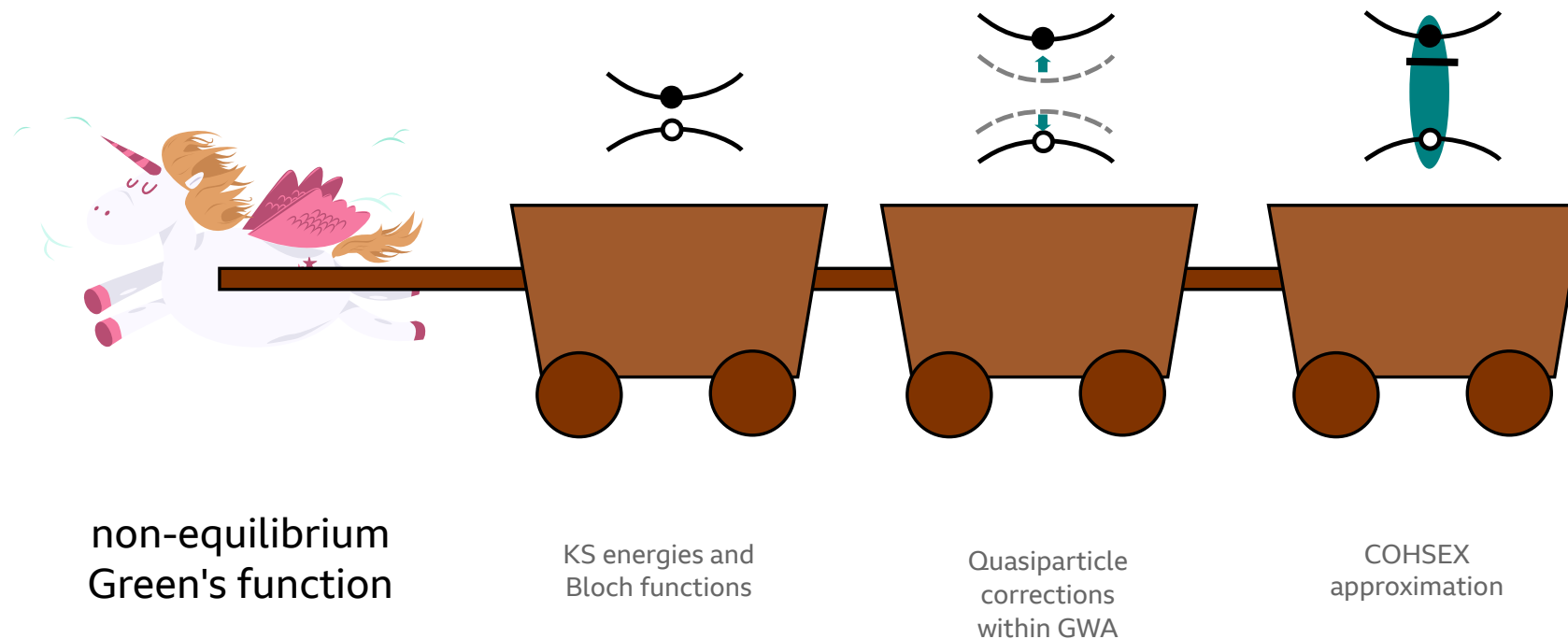
a paradigm shift

real-time

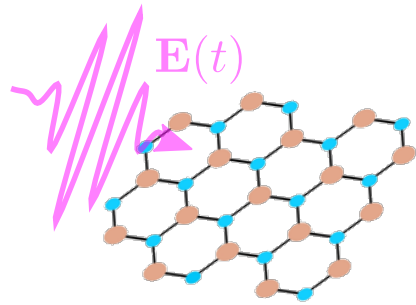
the GW+BSE recipe for success



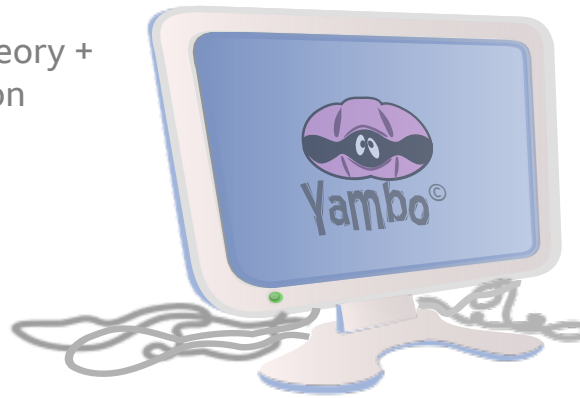
the GW+BSE recipe for success in real-time



yambo real-time implementation



Density functional theory +
GW approximation



off-diagonal term: polarization

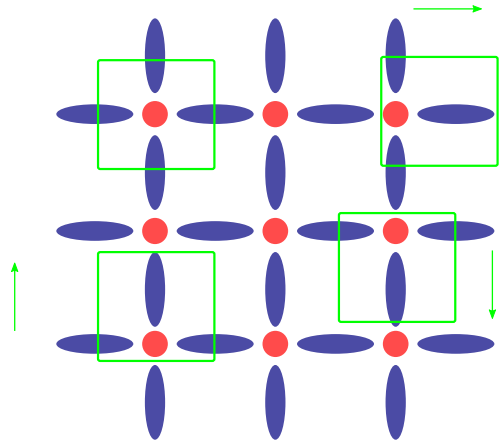
$$\mathbf{P}(t)$$

on diagonal terms: occupations

$$f_{n\mathbf{k}} = -iG_{nn\mathbf{k}}^<$$

$$i\hbar \frac{\partial}{\partial t} G_{nm\mathbf{k}}^<(t) = [\mathbf{h}_{\mathbf{k}} + \Delta\mathbf{h}_{\mathbf{k}} + \mathbf{U}_{\mathbf{k}} + \Delta\mathbf{V}_{\mathbf{k}}^H[\rho] + \Delta\Sigma_{\mathbf{k}}^{\text{cohsex}}[G^<], \mathbf{G}_{\mathbf{k}}^<(t)]_{nm}$$

bulk polarization

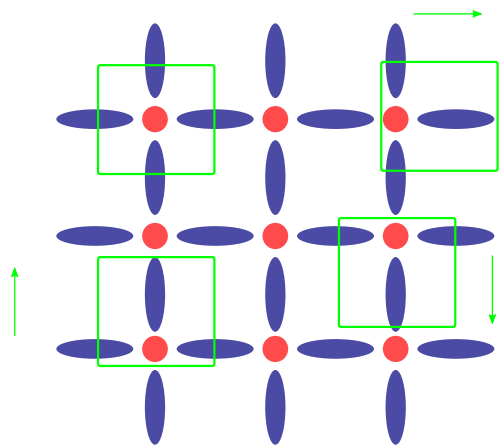


X $\Delta P = \int_{\text{space}} d\mathbf{r} \mathbf{r} \Delta\rho(\mathbf{r})$ *diverge!*

X $\Delta P \propto \int_{\text{BZ}} d\mathbf{r} \mathbf{r} \Delta\rho(\mathbf{r})$ *depends on cell choice*

see e.g. Resta, Troisieme Cycle de la Physique en Suisse Romande (1999),
"Berry's Phase and Geometric Quantum Distance"

bulk polarization



X $\Delta P = \int_{\text{space}} d\mathbf{r} \mathbf{r} \Delta \rho(\mathbf{r})$ diverge!

X $\Delta P \propto \int_{\text{BZ}} d\mathbf{r} \mathbf{r} \Delta \rho(\mathbf{r})$ depends on cell choice



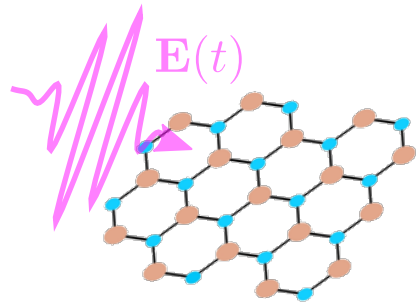
Polarization as a Berry-Phase

$$\Delta P \propto \langle X \rangle = \frac{L}{2\pi} \text{Im} \log \langle \Psi_0 | e^{i \frac{2\pi}{L} \hat{X}} | \Psi_0 \rangle \longrightarrow \propto i \int d\mathbf{k} \sum_m^{\text{occ}} \langle u_{\mathbf{k},m} | \partial_{\mathbf{k}} u_{\mathbf{k},m} \rangle$$

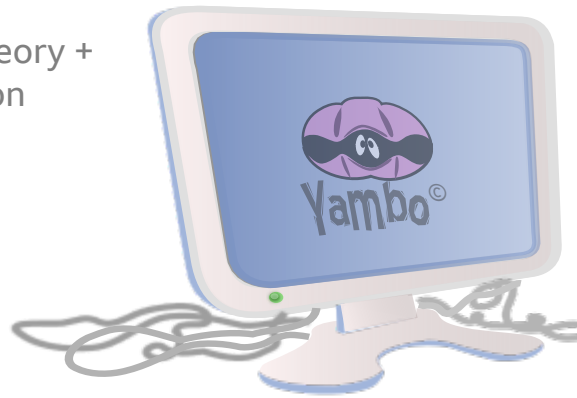
$\Psi_0 \approx$ Slater determinant

see e.g. Resta, Troisieme Cycle de la Physique en Suisse Romande (1999), "Berry's Phase and Geometric Quantum Distance"

dynamics of Bloch electrons for coherent response



Density functional theory +
GW approximation



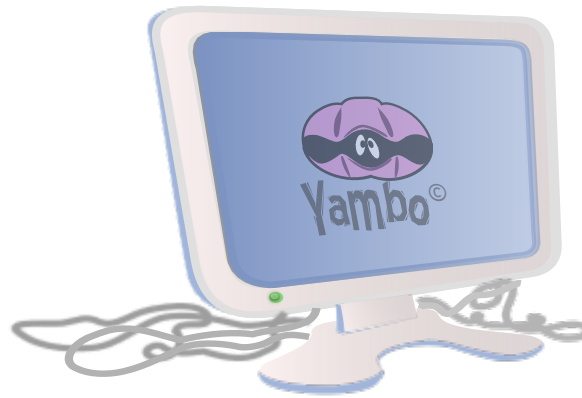
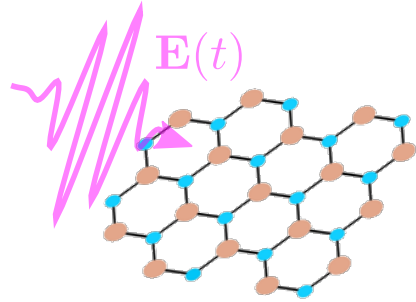
$$P(t) = i \frac{e}{V} \int d\mathbf{k} \sum_m^{\text{fill}} \langle v_{\mathbf{k},m} | \partial_{\mathbf{k}} v_{\mathbf{k},m} \rangle$$

$$i\hbar \frac{d}{dt} |v_{\mathbf{k},m}\rangle = (\mathbf{h}_{\mathbf{k}} + \Delta \mathbf{h}_{\mathbf{k}} + \Delta \mathbf{V}_{\mathbf{k}}^H[\rho] + \Delta \Sigma_{\mathbf{k}}^{\text{cohsex}}[G^<] + w_{\mathbf{k}}(\mathcal{E})) |v_{\mathbf{k},m}\rangle$$

w: covariant dipole operator, consistent with def of P(t)

$$G^<(\mathbf{r}, \mathbf{r}'; t) = i \int d\mathbf{k} \sum_m^{\text{fill}} e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')} v_{\mathbf{k},m}(\mathbf{r}; t) v_{\mathbf{k},m}^*(\mathbf{r}'; t)$$

real-time and nonlinear runlevels



yambo_rt

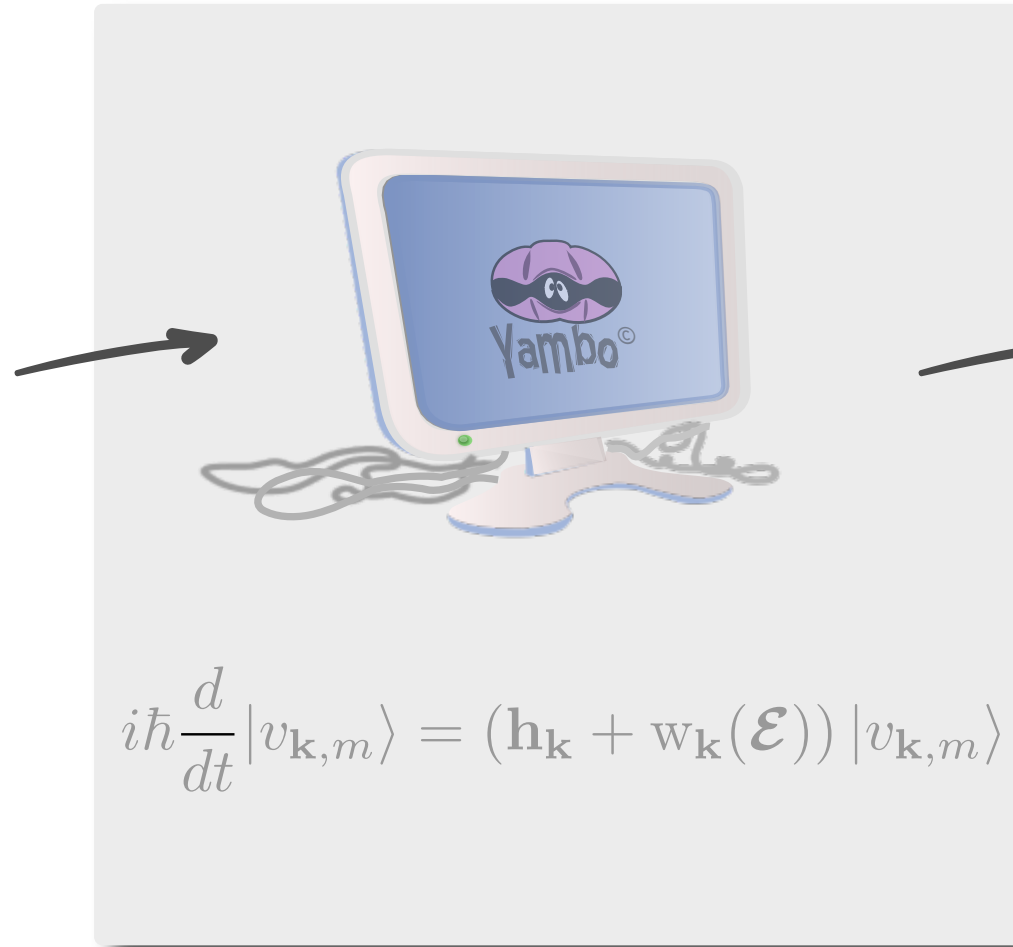
$$i\hbar \frac{\partial}{\partial t} G_{nm\mathbf{k}}^<(t) = [\mathbf{h}_{\mathbf{k}} + \Delta\mathbf{h}_{\mathbf{k}} + \mathbf{U}_{\mathbf{k}} + \Delta\mathbf{V}_{\mathbf{k}}^H[\rho] + \Delta\Sigma_{\mathbf{k}}^{\text{cohsex}}[G^<], \mathbf{G}_{\mathbf{k}}^<(t)]_{nm}$$

yambo_nl

$$i\hbar \frac{d}{dt} |v_{\mathbf{k},m}\rangle = (\mathbf{h}_{\mathbf{k}} + \Delta\mathbf{h}_{\mathbf{k}} + w_{\mathbf{k}}(\mathcal{E}) + \Delta\mathbf{V}_{\mathbf{k}}^H[\rho] + \Delta\Sigma_{\mathbf{k}}^{\text{cohsex}}[G^<]) |v_{\mathbf{k},m}\rangle$$

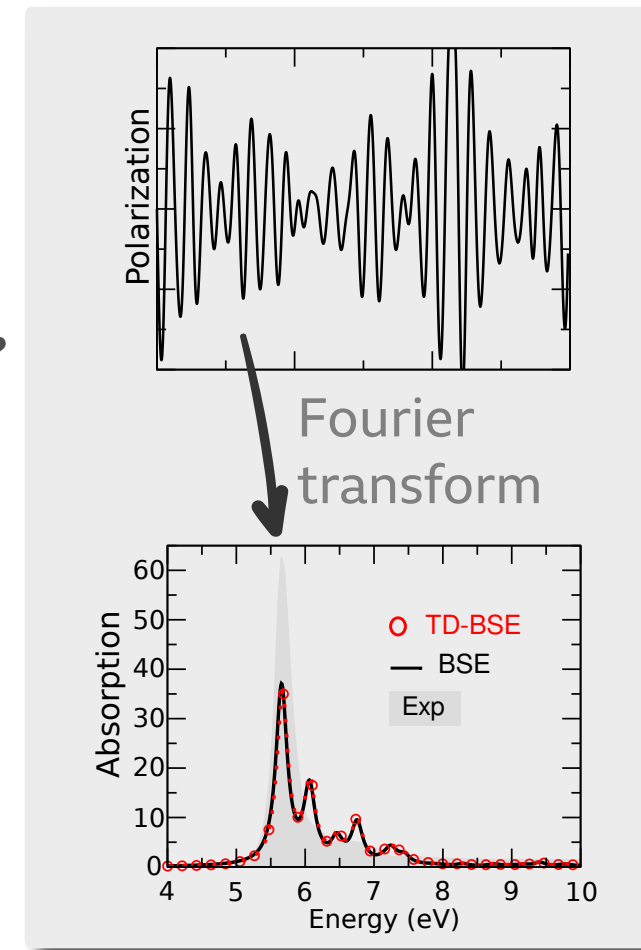
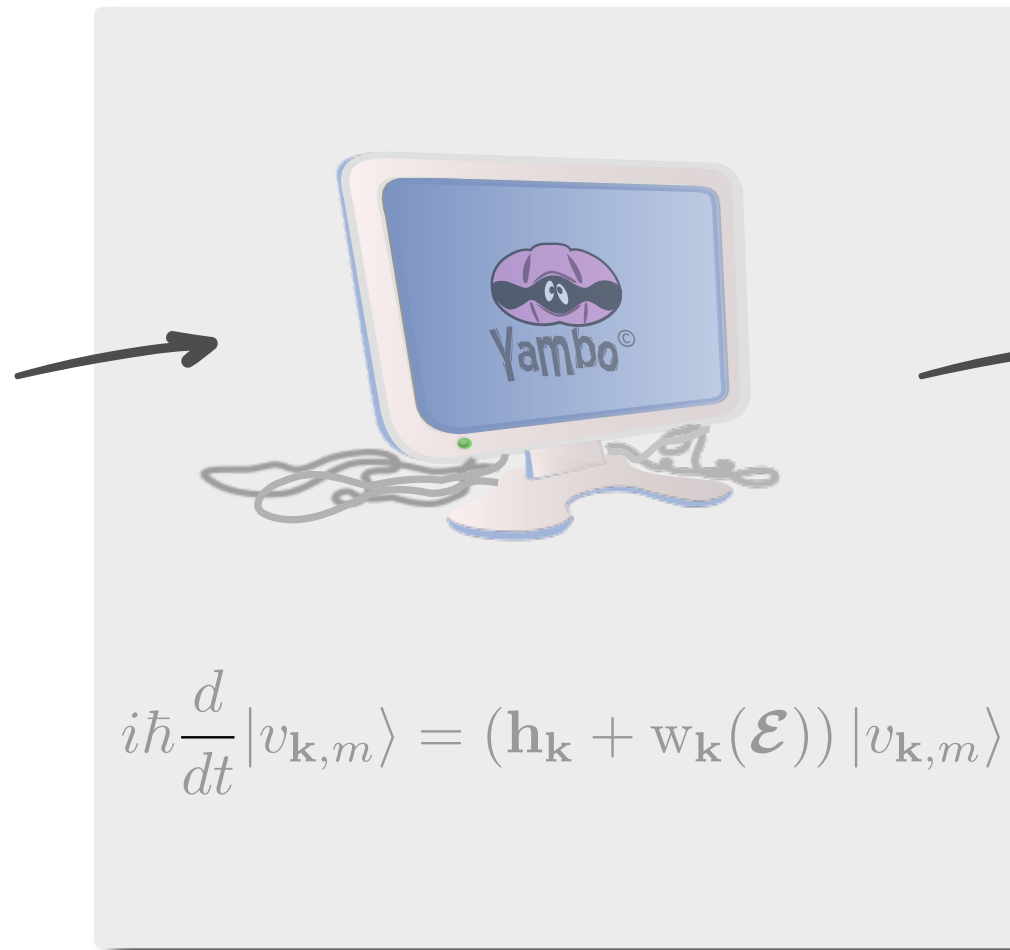
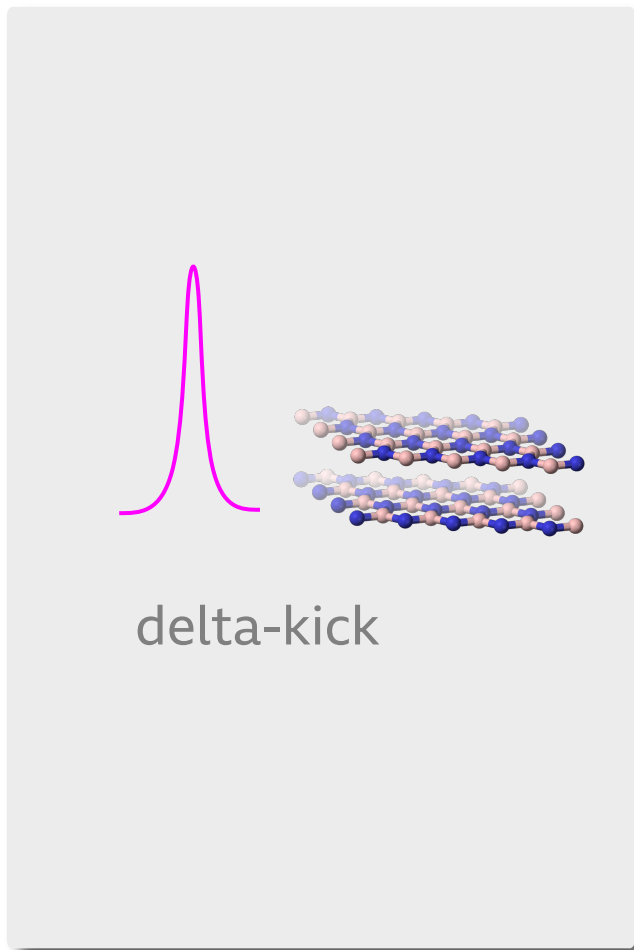
how it works? Choice of experiment

time-dependent
electric field

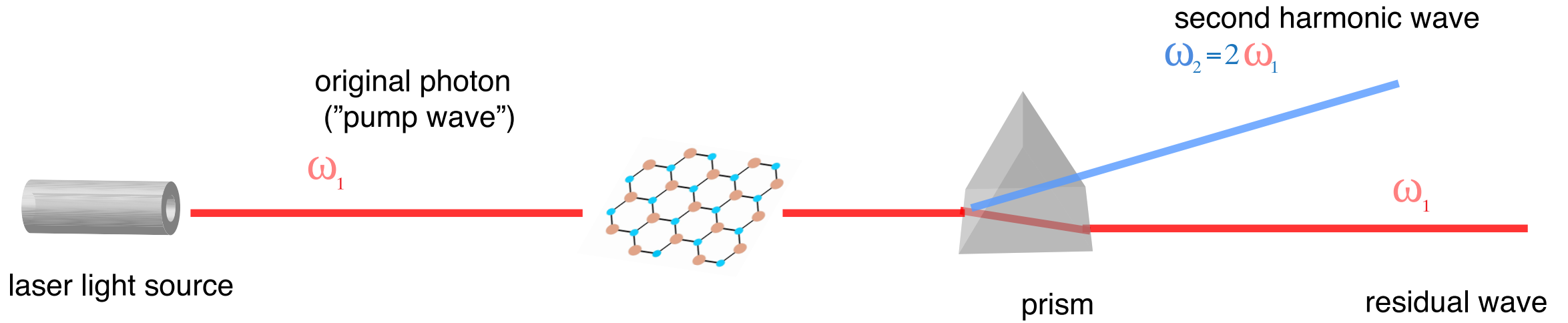


signal
post-processing


example: linear response

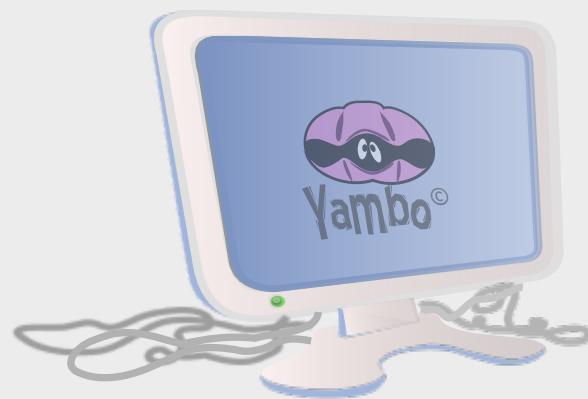


example: nth harmonic generation

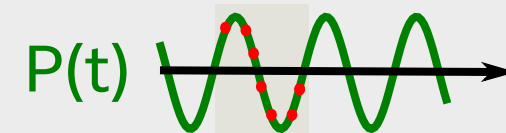


example: nth harmonic generation


$$\mathcal{E}(t) = \mathcal{E}_0 \sin(\omega_L t)$$



$$i\hbar \frac{d}{dt} |v_{\mathbf{k},m}\rangle = (\mathbf{h}_{\mathbf{k}} + \mathbf{w}_{\mathbf{k}}(\mathcal{E})) |v_{\mathbf{k},m}\rangle$$

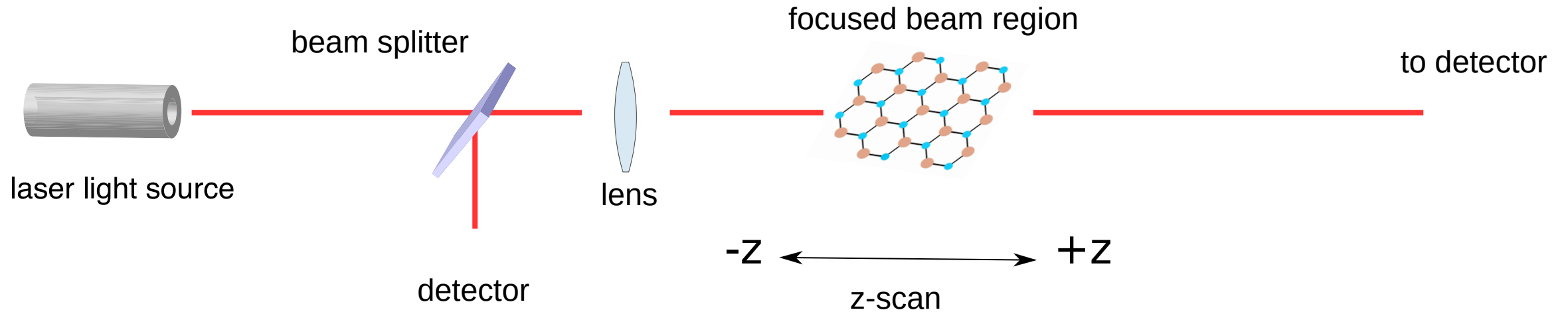


$$P(t_j) = \sum_{n=-N}^N F_{jn} \hat{P}_n \exp(in\omega_L t_j)$$


$$\hat{P}_n = \sum_{j=0}^{2N} F_{nj}^{-1} P(t_j)$$

$$\chi^{(n)}(n\omega_L) = \frac{\hat{P}_n}{\mathcal{E}^n}$$

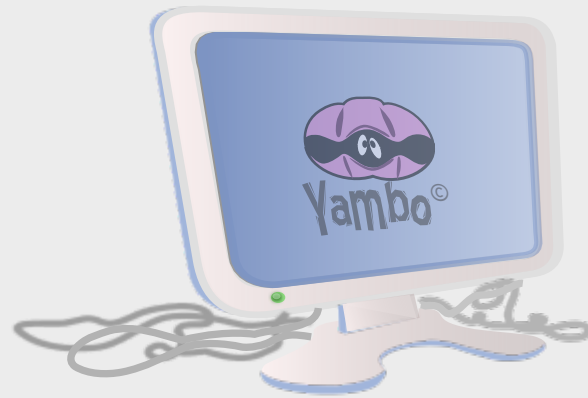
example: two-photon absorption



example: two-photon absorption

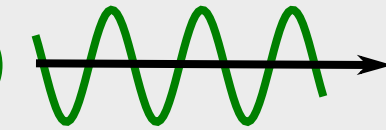

$$\mathcal{E}(t) = \mathcal{E}_0 \sin(\omega_L t)$$

@varying
field intensity



$$i\hbar \frac{d}{dt} |v_{\mathbf{k},m}\rangle = (\mathbf{h}_{\mathbf{k}} + \mathbf{w}_{\mathbf{k}}(\mathcal{E})) |v_{\mathbf{k},m}\rangle$$




$$P(t)$$




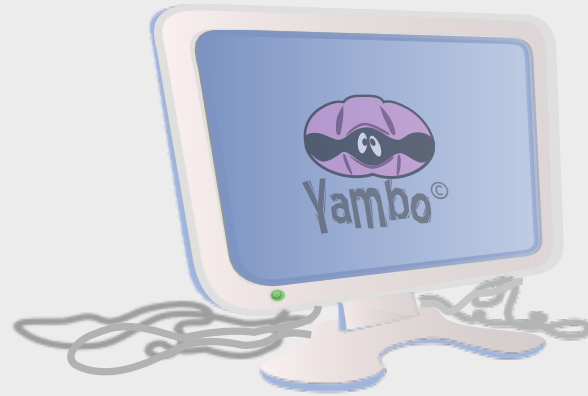
$$\begin{aligned} P(\mathcal{E}) &= \chi^{(1)} \mathcal{E} + \chi^{(3)} \mathcal{E}^3 + O(\mathcal{E}^5), \\ P\left(\frac{\mathcal{E}}{2}\right) &= \chi^{(1)} \frac{\mathcal{E}}{2} + \chi^{(3)} \frac{\mathcal{E}^3}{8} + O(\mathcal{E}^5), \\ P\left(\frac{\mathcal{E}}{4}\right) &= \chi^{(1)} \frac{\mathcal{E}}{4} + \chi^{(3)} \frac{\mathcal{E}^3}{64} + O(\mathcal{E}^5) \end{aligned}$$



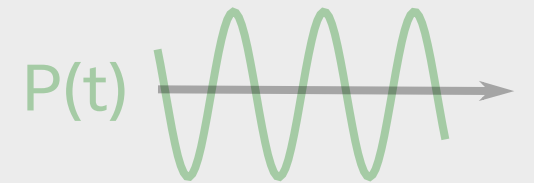
$$\chi^{(3)} = \frac{8}{3} \frac{P(\mathcal{E}) - 6P\left(\frac{\mathcal{E}}{2}\right) + 8P\left(\frac{\mathcal{E}}{4}\right)}{\mathcal{E}^3}$$

how it works? Choice of level of theory


$$\mathcal{E}(t) = \mathcal{E}_0 \sin(\omega_L t)$$

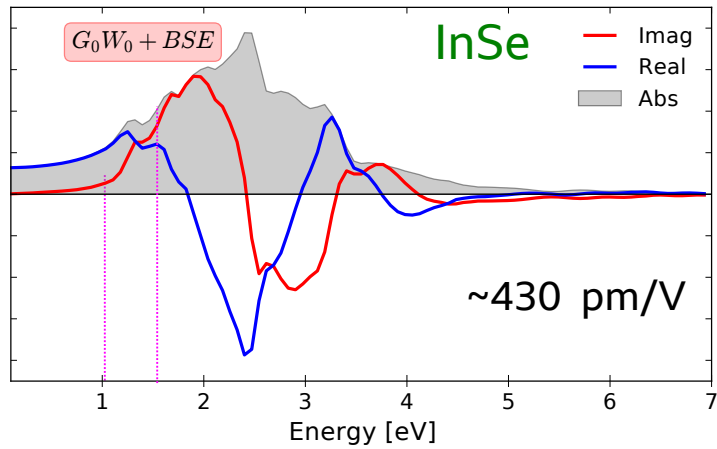


$$i\hbar \frac{d}{dt} |v_{\mathbf{k},m}\rangle = (\mathbf{h}_{\mathbf{k}} + \mathbf{w}_{\mathbf{k}}(\mathcal{E})) |v_{\mathbf{k},m}\rangle$$



non-linear optics at GW+BSE

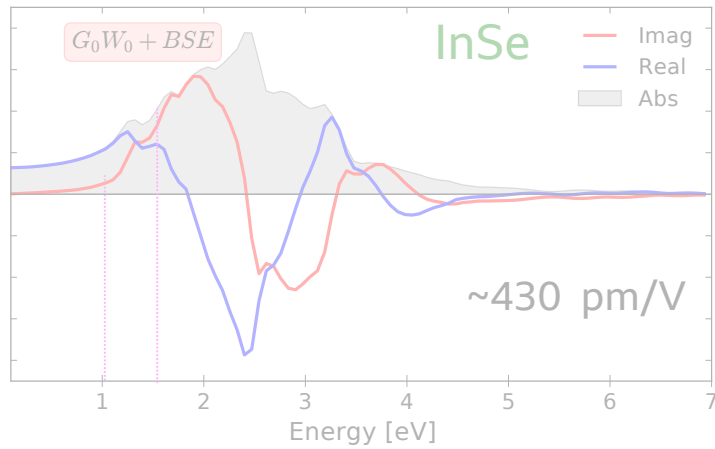
SHG in 2D materials



Phys. Rev. Mat. 3, 074003 (2019)

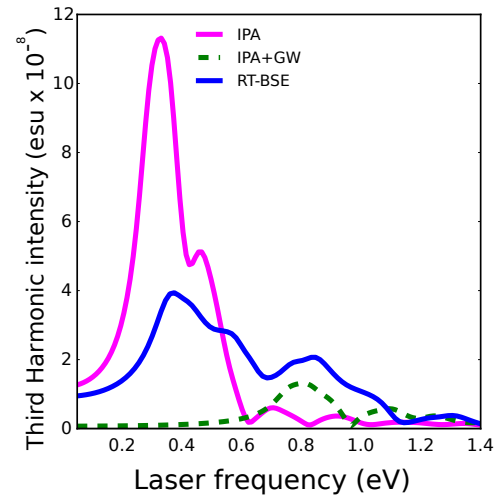
non-linear optics at GW+BSE

SHG in 2D materials



Phys. Rev. Mat. 3, 074003 (2019)

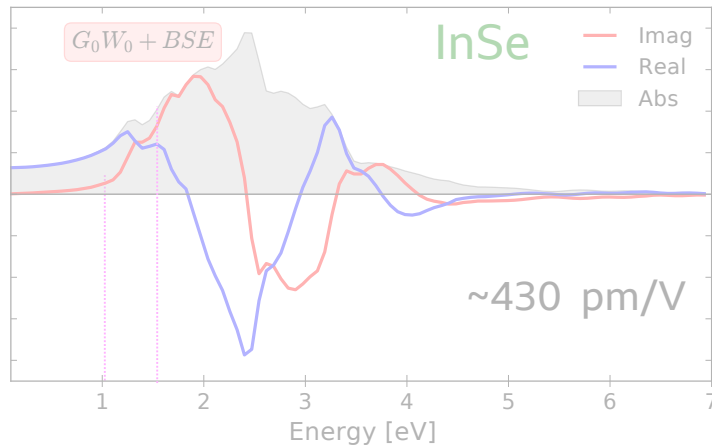
THG in 1D nanostructures



Phys. Rev. B 95, 125403 (2017)

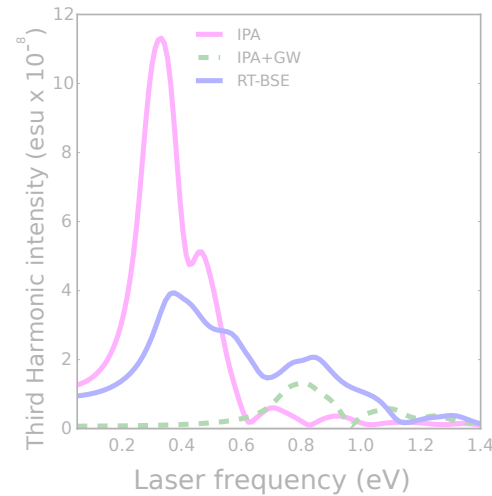
non-linear optics at GW+BSE

SHG in 2D materials



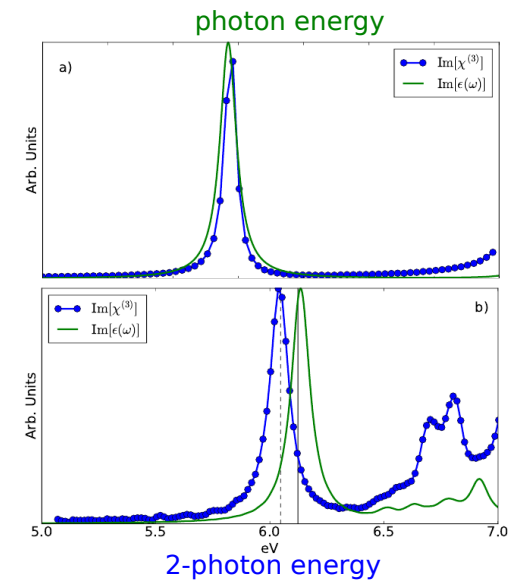
Phys. Rev. Mat. 3, 074003 (2019)

THG in 1D nanostructures



Phys. Rev. B 95, 125403 (2017)

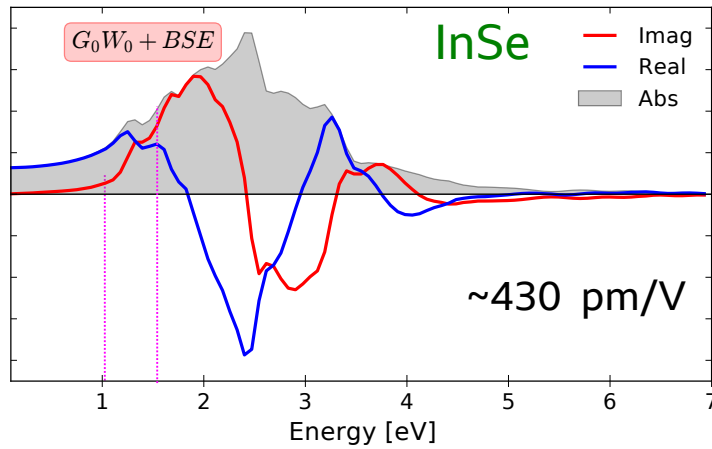
2-photon absorption in hBN bulk and ML



Phys. Rev. B 98, 1651126 (2018)

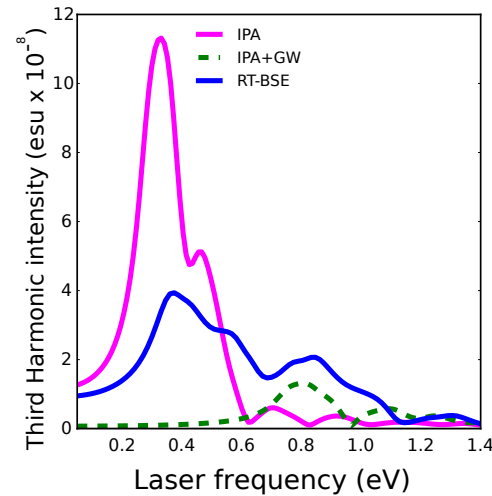
non-linear optics at GW+BSE

SHG in 2D materials



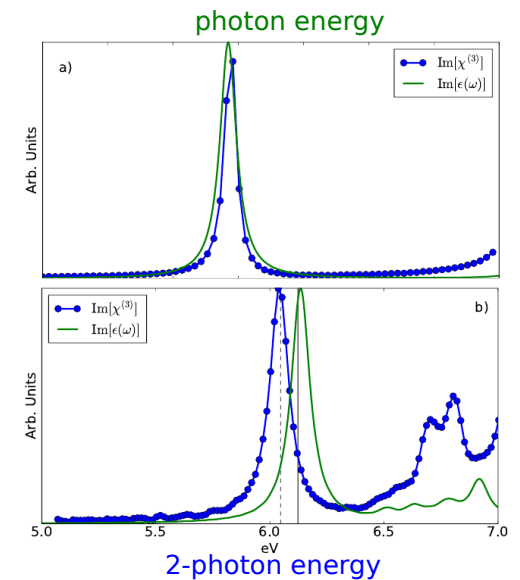
Phys. Rev. Mat. 3, 074003 (2019)

THG in 1D nanostructures



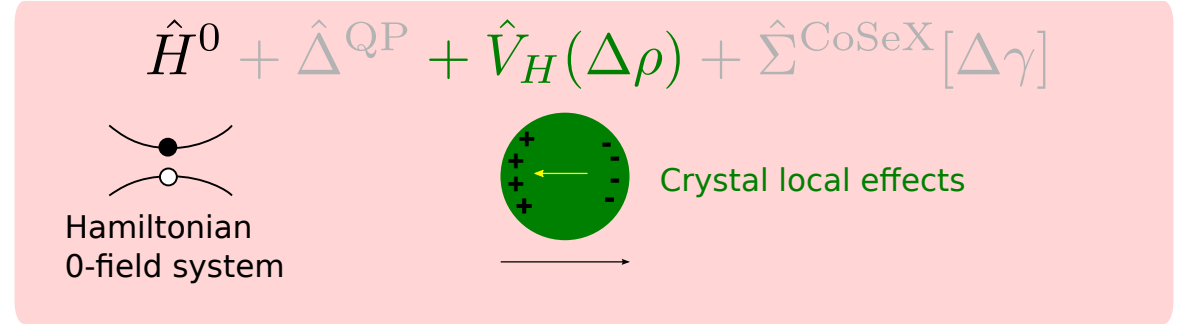
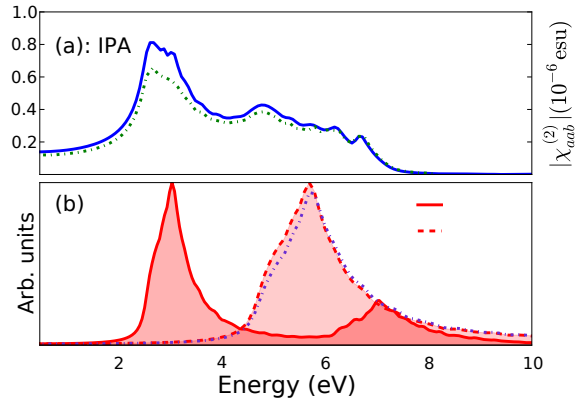
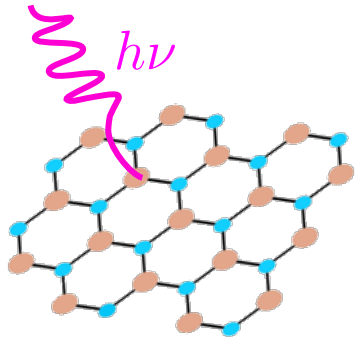
Phys. Rev. B 95, 125403 (2017)

2-photon absorption in hBN bulk and ML

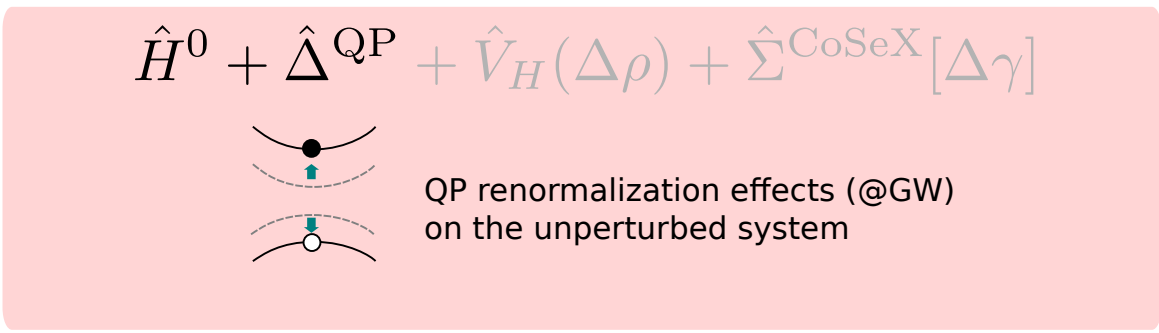
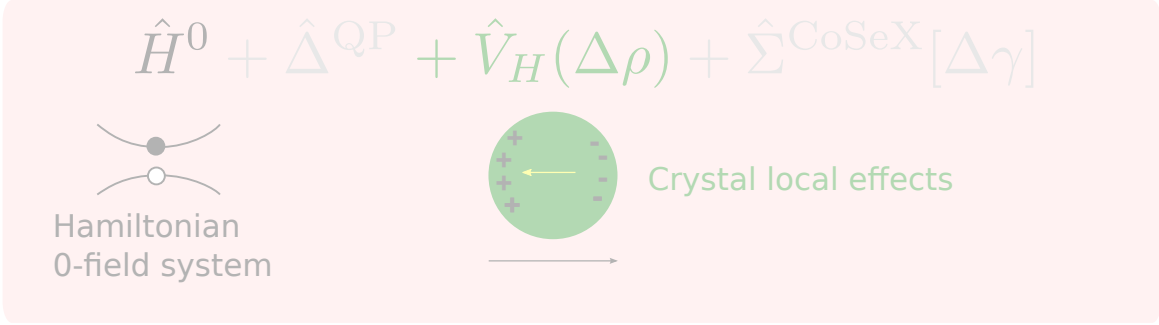
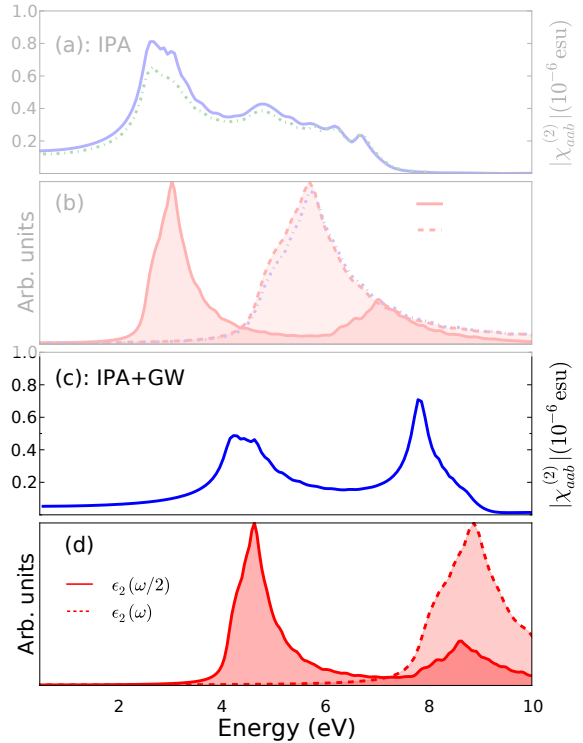
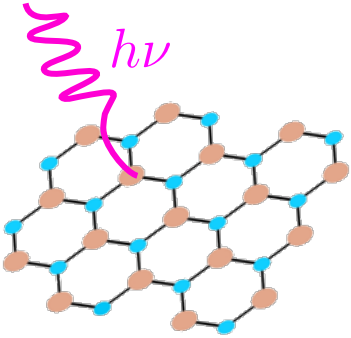


Phys. Rev. B 98, 1651126 (2018)

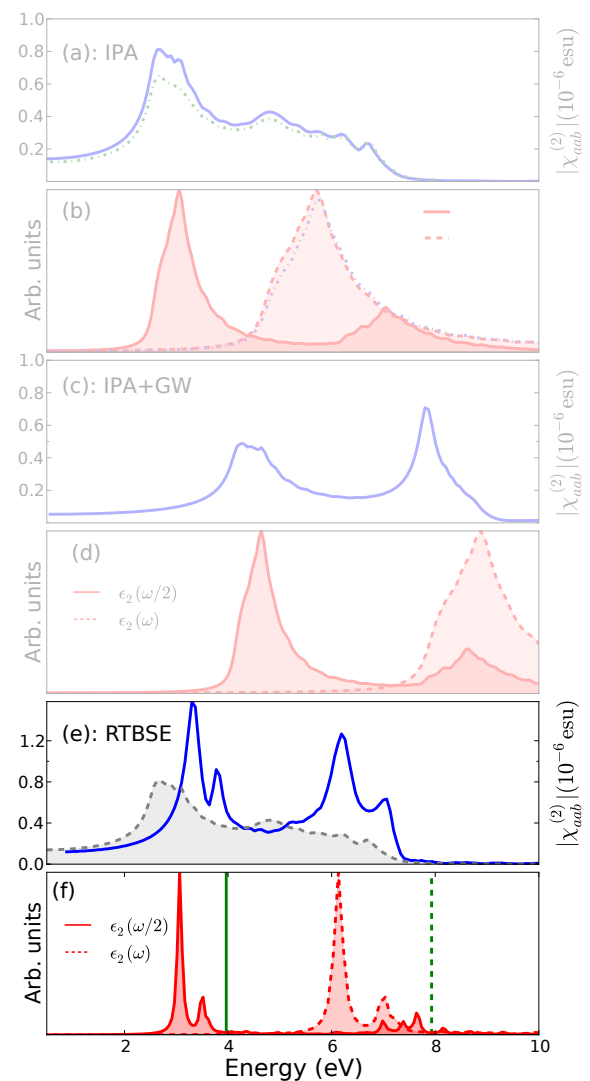
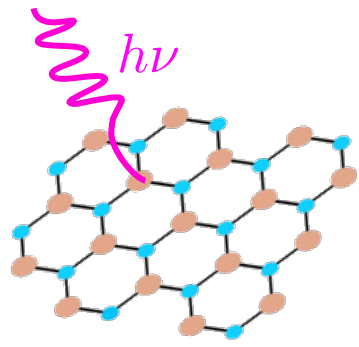
example: SHG in h-BN monolayer



example: SHG in h-BN monolayer



example: SHG in h-BN monolayer



$\hat{H}^0 + \hat{\Delta}^{QP} + \hat{V}_H(\Delta\rho) + \hat{\Sigma}^{CoSeX}[\Delta\gamma]$

Hamiltonian 0-field system Crystal local effects

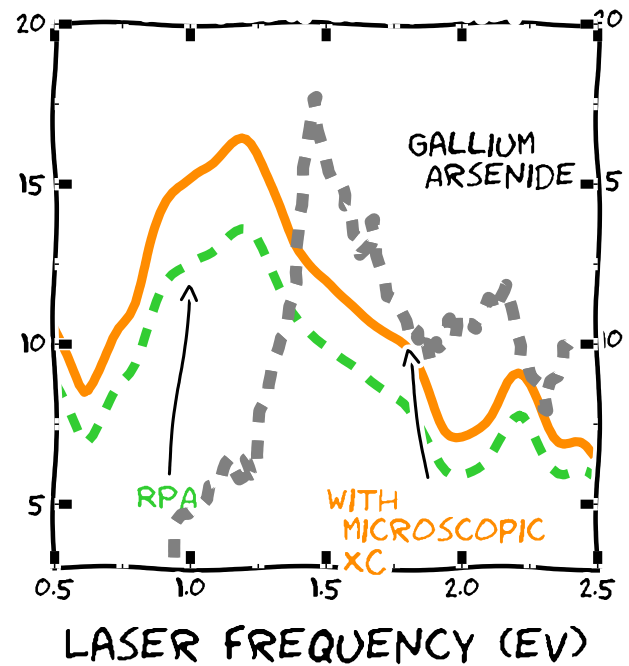
$\hat{H}^0 + \hat{\Delta}^{QP} + \hat{V}_H(\Delta\rho) + \hat{\Sigma}^{CoSeX}[\Delta\gamma]$

QP renormalization effects (@GW) on the unperturbed system

$\hat{H}^0 + \hat{\Delta}^{QP} + \hat{V}_H(\Delta\rho) + \hat{\Sigma}^{CoSeX}[\Delta\gamma]$

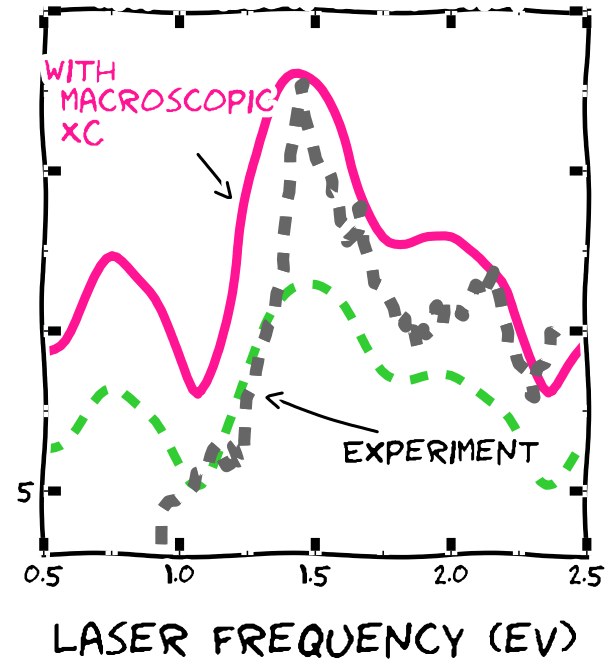
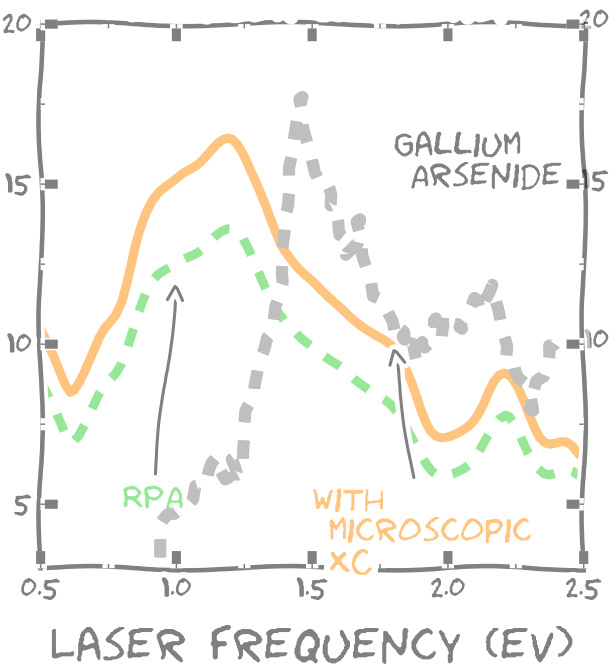
TD renormalization effects on QP energies/optical excitations

example: SHG in bulk semiconductors



$$H^0 + \Delta V^H[\rho] + \Delta V^{xc}[\rho]$$

example: SHG in bulk semiconductors



$$H^0 + \Delta V^H[\rho] + \Delta V^{xc}[\rho]$$

$$H^0 + \Delta^{\text{scissor}} + \Delta V^H[\rho] + \alpha^{xc} P$$

MG, D. Sangalli, C. Attaccalite PRB 94, 035149 (2016))



DRIVING THE EXASCALE TRANSITION

Follow us on:

 [company/max-centre/](https://www.linkedin.com/company/max-centre/)

 [@max_center2](https://twitter.com/max_center2)

 <http://www.max-centre.eu/>

THANKS